

# Living Math!

"One can invent mathematics without knowing much of its history.  
One can use mathematics without knowing much, if any of its history.  
But one cannot have a mature appreciation of mathematics without a  
substantial knowledge of its history." Abe Schenitzer, *Five Fingers to Infinity*.

## LIVING MATH THROUGH HISTORY LESSON PLANS

### Cycle 1, Unit 1 (C1U1)

#### Advanced and High School Levels **SAMPLE**

#### Early Cultures Through the Classical Age

Lesson 2: Ancient Roots of Mathematics in Africa and Mesopotamia

Lesson 3: Ancient Roots of Mathematics in Asia

Lesson 4: Ancient Roots of Mathematics in the Americas

Lesson 5: Thales and the Egyptians

Lesson 6: Pythagoras and the Early Greeks

Lesson 7: Archimedes' Geometry

Lesson 8: Eratosthenes, and Archimedes' Large Numbers

Review Activities

# Living Math Through History

## C1U1 Advanced and High School Level Lesson Plans - SAMPLE

### Early Cultures Through the Classical Age

If you are not familiar with the format of the Living Math lesson plans, refer to the Parent Introduction Letter for notes on how to use resources.

#### Living Math Lesson Plans Icon key



Best done with two or more people



Book resource in print and/or online



Highly visual resource, pictures




Printable handout



Hands on activity



Primary,  Intermediate or



Advanced level



Writing or drawing activity



Quote to think about or discuss



Discussion activity



Internet website



Calculator recommended



Video resource



Solutions

## Living Math Through History

### Thales, the Egyptians and Measurement - SAMPLE LESSON

Advanced and High School Levels - Lesson 5

**Synopsis** - Thales lived his life during an age in which few written works survived the ravages of time. We know much about him, however, because he was written of in multiple ancient accounts, and he was referred to as one of the "Seven Wise Men" or "Sages" of ancient times.

Thales is the first ancient known to use the process of observing patterns and deducing logical conclusions that could be applied to other problems. Reading about his life, we find several problems he tackled that are world famous: predicting crop production from weather cycles, measuring the height of the Great Pyramid (so great it covered 12 acres), and solving the entertaining "donkey and the salt mine" problem.

Thales made great contributions to mathematics and astronomy. He proved that if an angle is drawn in (inscribed) in a half circle, it will always be a right triangle. It's one thing to be able to observe something that appears to be true, but quite another matter to logically **prove** it will work every time.

It is not believed that Thales left any writings of his own, although there are a few works scholars debate may have been written by him. Most of what we know about Thales is from ancient writers such as Herodotus, Diogenes and Sosicrates. Because of the second and third hand nature of the sources, much of what we know about Thales is considered "anecdotal," based on stories handed down, of which we are not sure all the details are true. That Thales lived, however, and produced many highly original ideas in mathematics, science and even business is widely accepted to be true.

For more, visit: <http://www-groups.dcs.st-and.ac.uk/~history/Biographies/Thales.html>



#### **Joy of Mathematics** (*vignettes to inspire exploration*):

p. 36 Thales and the Great Pyramid    p. 93, Cannon Balls and Pyramids  
p. 119 The Penny Puzzle                      p. 214 Triangular, Square, Pentagonal Numbers



#### **Math Talk Poetry in Two Voices:** "Triangles" pp. 41-42



#### **Math History Readers**, one or more as desired:

**String, Straightedge & Shadow** by Julia Diggins pp. 58 - 91


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**Euclid's Window** by Leonard Mlodinow - more advanced reading than SS&S, beginning through Section 1.3

**The Pantheon Story of Mathematics for Young People** by James T. Rogers - pp. 29-33. An older style book, well illustrated, middle school level.


**The Story of Science**  by Joy Hakim, Chapters 4, 5 and 6. These chapters are highly relevant to mathematics history


  **Pyramid and Unbuilding** by David Macaulay and/or **Kingfisher World of Pyramids** (all are excellent)


 Optional Historical Context - **The Egyptian News** by Scott Steedman (fun tabloid-style book)

#### High School Mathematics and History Reading

 **Euclid's Window** by Leonard Mlodinow - Beginning through Section 1.3

 **HS+ From Five Fingers to Infinity, A Journey Through the History of Mathematics** - *Recommended Reading: Essays 21 - 24* (p. 133 - 154) The Rhind Papyrus, Volume of a Truncated Pyramid, Practicality of the Rule of False Position, The Dawn of Demonstrative Geometry. See introduction on how to use this resource, some of the chapters are more technical than others.

 **HS+ Mathematics: From the Birth of Numbers** - 2.1 - Egyptian Numeration, 2.3 Positional Notation; 9.01 History - Rhind / Ahmes Papyrus portions only

 **History of Science: Antiquity to 1700** (Teaching Company) Lecture 3, the Pre-Socratics - College level series

#### **Mathematical Ideas associated with Thales:**

Measurement

Geometry of triangles

Problem Solving

Area, perimeter

Egyptian multiplication

Mathematical Reasoning

# Living Math Through History


## Thales, the Egyptians and Measurement - SAMPLE LESSON

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### Vocabulary related to this lesson:


Theorem	Proof	Deductive	Inductive	Reasoning
Philosopher	Scientist	Ratio	Proportion	Prediction

### Quotes to think about and discuss:


 "Know Thyself." Thales is given credit for this quote, and for being one of the Seven Sages. What is a "Sage"?

Why do you think this quote would be passed down for thousands of years? What does it have to do with mathematics?

Do you think you are on the road to "knowing yourself"?

 "Mathematics is not a deductive science - that's a cliché. When you try to prove a theorem, you don't just list the hypotheses, and then start to reason. What you do is trial and error, experimentation, guesswork." -- Paul Halmos

[Halmos](#) was a mathematician of this century who only recently passed away, in 2006, having lived during some of the most exciting developments in math and science in history. Does this match up with your impression of mathematics?

 "[Sesostris] divided the land into lots and gave everyone a square piece of equal size, from the produce of which he exacted an annual tax." Herodotus

Sesostris was king of Egypt approximately 1322 BC. Herodotus, a Greek known as the "Father of History," wrote the first account of history about 900 years later.

What kind of mathematics would the Egyptians need in order to do this division fairly? What kind of mathematics would they need to compute annual taxes?

### Discussion or journaling questions


**Thales** was considered a **philosopher**, **mathematician** and a **scientist**. How did Thales' life embody the best of all of these in one individual?


Thales' story is the first specific example of mathematical ideas being exchanged through one individual's travels. He learned geometry from the Egyptians and astronomical science from the Babylonians that he took back to his native Asia Minor.


## Living Math Through History


### Thales, the Egyptians and Measurement - SAMPLE LESSON


Advanced and High School Levels - Lesson 5

 Why were the Egyptians such good geometers (literally, "earth measurers")? You might want to revisit this question after doing the math activities.


 What is the **Rhind** or **Ahmes Papyri**? Why is this so interesting to the study of mathematics?

 The Egyptian physician **Troth** devised a solar calendar known as the **Calendar of Troth** - how was this constructed?

 **Geography and Timeline:** When reading about Thales, follow his path through his birthplace of Miletus (a Greek colony in what is now Turkey / Asia Minor) through Babylonia and Egypt, and correspond these with the modern day countries in these regions.

 **What Was Happening When . . .** What major event do ancient writings say that Thales predicted in 585 B.C.?

### Mathematical Activities

 Thales is known for proving several geometric theorems. See if you can support his theorems with logic, experimentation, and intuition.

- \* A circle is bisected by any diameter
- \* The base angles of an isosceles triangle are equal
- \* If two lines intersect the vertical angles formed are equal
- \* Two triangles are equivalent if they have two angles and one side in each respectively equal
- \* An angle inscribed in a semicircle is a right angle.


 Websites recommended for **Egyptian multiplication**, this is quite interesting! <http://atozteacherstuff.com/pages/296.shtml> explains how to do it.

<http://mathforum.org/library/drmath/view/57542.html> is another example. No multiplying needed to multiply! It is related to binary numbers, can you explain why it works?


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 If you observe the world around you, you will notice that triangles are everywhere. This is particularly true of structures - buildings, bridges, towers - all are supported by triangles. Since Thales and the Egyptian pyramids were a focal point of the reading, seeing the structure in David Macaulay's books, **Pyramid** and **Unbuilding** is suggested. These books show the insides of structures, and you can see that triangles are built into all structures, even seemingly square or rectangular structures.

Find or create several forms of squares, cubes, triangles and pyramids. These can be constructed from Zome Tools (<http://www.zometool.com/>), K'nex or other building sets. Use the forms to show the strength of the triangle structure compared to the square. This is an excellent way to demonstrate why triangles are engineering marvels. See the Straw Tower project below, a great extension activity.

 A **Straw Tower project** is a fantastic activity, particularly if you do a complete activity including designing and testing the model. We have done this several times with a group and had competitions. Supplies are very inexpensive, all that is required is straws, paperclips and tape.

Here is one version of this project, it is best done in a group where different designs can be tested, so that you can see which designs hold the most weight. I suggest if you like this project that you save a copy of the plans.

<http://www.ces.ncsu.edu/depts/fourh/old/greenlight/afterschool/StrawTowers.pdf>

 **Rhind Papyrus math:** See if you can solve the following math problems from the Rhind Papyrus. The hardest part is following the wording, the math isn't that difficult.

- 1) A quantity and its  $\frac{1}{5}$  added together become 21. What is the quantity?
- 2) A quantity, its  $\frac{1}{2}$ , and its  $\frac{1}{4}$  added together become 10. What is the quantity?
- 3) How many cattle are there in a herd when  $\frac{2}{3}$  of  $\frac{1}{3}$  of them make 70, the number due as tribute to the owner?
- 4) Sum the geometrical progression of five terms of which the first term is 7 and the multiplier is 7.

Solutions at the end of this section.

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#### Rhind Papyrus Challenge Problem 62



5) A bag containing equal weights of gold, silver, and lead has been bought for 84 *sha'ty* (unit of value). What is the amount in the bag of each precious metal if a *deben* (unit of weight) of gold costs 12 *sha'ty*, a *deben* of silver costs 6 *sha'ty*, and a *deben* of lead costs 3 *sha'ty*?

6) *Rhind Papyrus Challenge Question*: The area of a circle is described as being equal to the area of a square with a side that is  $\frac{8}{9}$  of the diameter. What does this yield as a value for pi?

**Thales' Method** [http://www.chatham.edu/PTI/Math\\_Connection/Renne\\_01.htm](http://www.chatham.edu/PTI/Math_Connection/Renne_01.htm) Based on a story of Thales in one of his many trips to Egypt. Upon reaching the great pyramid of Giza (Khufu), he asked the guide what its height was. While the Guide was asking some scribes, because he did not know the answer, Thales calculated the height of the pyramid. He had measured the pyramid's shadow as well as his own. Then by calculating the ratio of his own height to his own shadow, he was able to calculate the height of the pyramid.

If Thales was 1.6 meters tall and cast a shadow that is 2.3 meters at the same time of day that the Pyramid of Giza casts a 210-meter shadow, what is the height of the pyramid?

Would Thales really be able to measure the distance from the edge of the pyramid's shadow to the point where the height was perpendicular to the base, which is inside the pyramid? If the story is true, perhaps Thales did more math than was implied. Many of the stories about Thales vary in details when compared to different sources, which is why we refer to these as anecdotes.

 **Pyramid math activity** - see Living Math . Accuracy counts with this activity! Calculator usage is recommended for this activity, you will be measuring and calculating in tenths of centimeters, you don't want to spend hours doing manual arithmetic, especially since a number of the results may involve repeating decimals.



Remember, "garbage in, garbage out" - the calculator is only accurate if the input is accurate.




## Living Math Through History



### Thales, the Egyptians and Measurement - SAMPLE LESSON



Advanced and High School Levels - Lesson 5



  Deductive reasoning requires logic. See how you do with these deductive reasoning logic puzzles, these are printable handouts

[http://www.abcteach.com/free/l/logic\\_mysterymathball.pdf](http://www.abcteach.com/free/l/logic_mysterymathball.pdf)

 **Stock Market Simulation** - If you are participating in a stock market simulation, continue to explore the math involved. Concepts include rounding to allow enough cushion for commissions and stock prices fluctuations, estimating, and mental vs. written arithmetic. See Lesson 3 for more information about the simulation.

  **Famous Problems and Their Mathematicians** by Art Johnson. p. 84 - 85 Head in the Clouds - geometry / algebra activity with background on Thales.

  **Algebra Activities from Many Cultures** by Beatrice Lumpkin, portions available online at <http://books.google.com/books> or for purchase. Activities 6: Egyptian multiplication, Activities 9, 10, and 11

  **Historical Connections In Mathematics Vol. 2** by Wilbert and Luetta Reimer, Chapter 1 contains handout activities. While the content will differ, \* indicates there is some overlap with Living Math Handouts, or that the same puzzle is available online.



Smart Shadows, \* an outdoor version of the Shadow Math Living Math activity.

Stars Around the Moon\*, this is also known as the Petals Around the Rose puzzle

Puzzling Pyramids


Dominoes on a Checkerboard \*

Who's Who on the Baseball Team?

  **Math Activities: Agnesi to Zeno**, Over 100 Vignettes from the History of Math by Sanderson Smith:

#5 The Rhind Papyrus and the St. Ives Puzzle p. 9

#9 Thales: A Man of Legend p. 17

 More activities and information may be found on the Thales page at Living Math <http://www.livingmath.net/> Math Sites: C1U1 Ancients and Cultures.

## Living Math Through History

### Thales, the Egyptians and Measurement - SAMPLE LESSON

#### Advanced and High School Levels - Lesson 5

✓ Solutions to Rhind Papyrus questions:

1)  $17-1/2$

2)  $5-5/7$

3) 315

4) 19,607

Challenge questions:

5) The sum of 12, 6 and 3 sha'ty for 1 deben of gold, 1 deben of silver, and 1 deben of lead is 21 sha'ty.

6) 3.160493827



#### Living Math Handouts

These are for **IA** levels unless otherwise indicated.

Great Pyramid Math instructions and templates

Shadow Math - includes instructions to bring the math down to **P**+

Think Like Thales - Leader Notes and Student Handout developed for a two-hour math group teaching unit, adaptable to home use. Heavy math focus in these activities.